$$AQ_0=AB\sin\theta=2AB\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta=2^2AB\sin(\theta/2^2)\cos(\theta/2^2)\cos\tfrac{1}{2}\theta$$
$$=2^nAB\sin(\theta/2^n)\cos\tfrac{1}{2}\theta\cos(\theta/2^2)\ldots\cos(\theta/2^n).$$
$$AQ_n=AB\sin(\theta/2^n).$$
$$\therefore (AQ_0/AQ_n)=2^n\cos\tfrac{1}{2}\theta\cos(\theta/2^2)\ldots\cos(\theta/2^n).$$
$$BQ_1.BQ_2.BQ_3\ldots BQ_n=(AB^n)\cos\tfrac{1}{2}\theta\cos(\theta/2^2)\ldots\cos(\theta/2^n)$$
$$=(\tfrac{1}{2}AB)^n(AQ_0/AQ_n)=(AO)^n(AQ_0/AQ_n).$$

CALCULUS.

78. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Investigate value of $\left(\frac{\tan x}{x}\right)^{1/x^n}$ where x is 0 and n has consecutive values 1, 2, 3, 4, ... Is there any law governing the different results? When $n=1$, result is 1; when $n=2$, result is $e^{\frac{1}{3}}$; $n=3$, gives ∞, etc.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

$$\left(\frac{\tan x}{x}\right)^{1/x^n}=e^{(1/x^n)\log[(\tan x)/x]}=y.$$

Limit of $\dfrac{\log\tan x-\log x}{x^n}$=limit of $\dfrac{\cot x\sec^2x-(1/x)}{nx^{n-1}}$=limit of $\dfrac{2x-\sin 2x}{nx^n\sin 2x}$,

but $\sin 2x=2x-\dfrac{8x^3}{6}+\dfrac{32x^5}{120}-\dfrac{128x^7}{5040}$, etc., $=2x-\dfrac{4x^3}{3}+\dfrac{4x^5}{15}-\dfrac{4x^7}{45}+$, etc.

$$\therefore \frac{2x-\sin 2x}{nx^n\sin 2x}=\frac{2x-2x+\dfrac{4x^3}{3}-\dfrac{4x^5}{15}+\dfrac{4x^7}{45}-\text{, etc.}}{nx^{n+1}(2-\dfrac{4x^2}{3}+\dfrac{4x^4}{15}-\dfrac{4x^6}{45}+\text{, etc.}}=\frac{30-6x^2+2x^4}{nx^{n-2}(45-30x^2+6x^4)},\ \text{ap-}$$

$$\text{proximately, }=\frac{2}{3nx^{n-2}}+\frac{14}{45nx^{n-4}}+\text{, etc., }=S.$$

When $n=1$, $S=0$ for $x=0$.
When $n=2$, $S=\frac{1}{3}$ for $x=0$.
When $n=3$, 4, 5, etc., $S=\infty$ for $x=0$.
$\therefore$ When $n=1$, $y=e^0=1$.
When $n=2$, $y=e^{\frac{1}{3}}$.
When $n=3$, 4, 5, etc., $y=e^{\infty}=\infty$.

Also solved by *ELMER SCHUYLER*, whose solution has been accidentally misplaced, and hence does not appear in this issue.